

First Steps Towards Modelling Expressive Timing in German Late Romantic Organ Music

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Research idea:

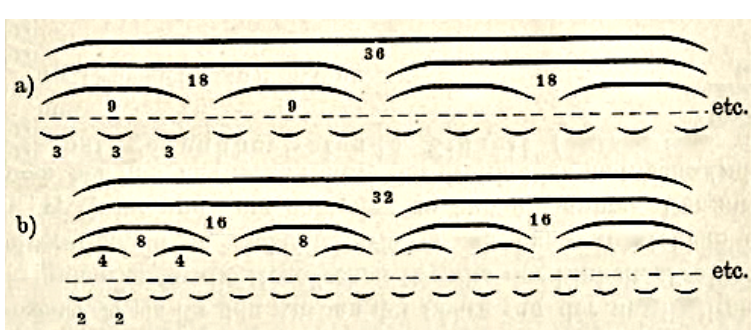
- ✓ Build a mathematical model for the symmetric phrasing scheme of German musicologist Hugo Riemann (1849–1919)
- ✓ Use it to create an artificially expressive timing pattern in organ works of Max Reger (1873–1916)

I. Introducing Hugo Riemann

- ❖ Hugo Riemann was the dominating music scholar of the XIXth century; he collected the most important trends of his time and abstracted them into strong scientific theories.
- ❖ He was a primary composition teacher of Max Reger, the “greatest German organ composer of the late Romantic period” [1].
- ❖ The influence of Riemann’s theory to the organ music Max Reger’s was noticed by several leading researchers in organ performance [2–3].

II. Mathematical interpretation of Riemann’s phrasing principles

Riemann’s phrasing scheme: the short 3-notes or 2-notes initial motives form larger groups in the symmetric hierarchical order [4]:



Mathematical model: phrasing arcs on each level are simulated as positive semi-ellipses:

$$y_{ij} = \left(\sqrt{1 - \left(x - \frac{h_{ij}}{a_{ij}}\right)^2} \right) * b_{ij} + T,$$

- i - number of the level, j - sequence number of the ellipse on the i^{th} level;
- a_{ij} - the long axe of ellipse, corresponds to the Riemannian motivic length;
- h_{ij} - x -coordinate of the ellipse’s center, corresponds to the middle point of each motive;
- T - starting metronomic tempo value (constant);
- b_{ij} - the short axe of ellipse, proportional to T :
 $b_{ij} = e_{ij} * T$

The parameter e is defined as **temporal elasticity**: it shows the maximum of the model tempo deviation against the metronomic tempo for each level.

Research goal: finding parameters e_{ij} that, on the one hand, would preserve the Riemannian idea of the built-in motivic symmetry, and on the other hand, would approximate the real performance data and therefore might be used in computer simulation of expressive timing.

III. Symmetric model

- The model was evaluated analytically on the Max Reger’s *Choral Prelude* op. 135a/1.
- Performance data was collected as the MIDI recording of the professional organist interpretation at the Casavant organ in the Church of Saint Andrew and Saint Paul (Montreal, Canada).
- Temporal information was extracted through the manual beat-mapping process in *Logic Pro X* and exported to *Matlab*.



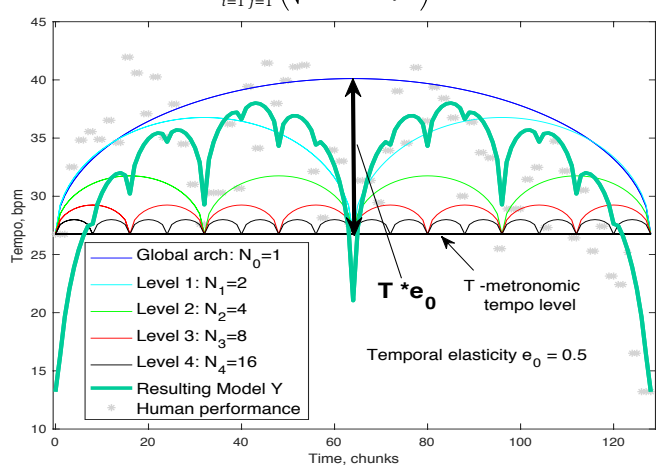
Symmetric model:

$$e_{ij} = e_i = 1.5 * e_0 / N_i,$$

- e_0 - global arch over the whole piece;
- e_{ij} - temporal elasticities on subsequent levels;
- N_i - overall quantity of ellipses on the i^{th} level.

Model tempo curve Y involving the global arch and 4 subsequent levels:

$$Y = Y_0 + \sum_{i=1}^4 \sum_{j=1}^{N_i} \left(\sqrt{1 - \left(x - \frac{h_{ij}}{a_{ij}}\right)^2} \right) * e_{ij} * T$$



IV. Weights optimization

Generic model:

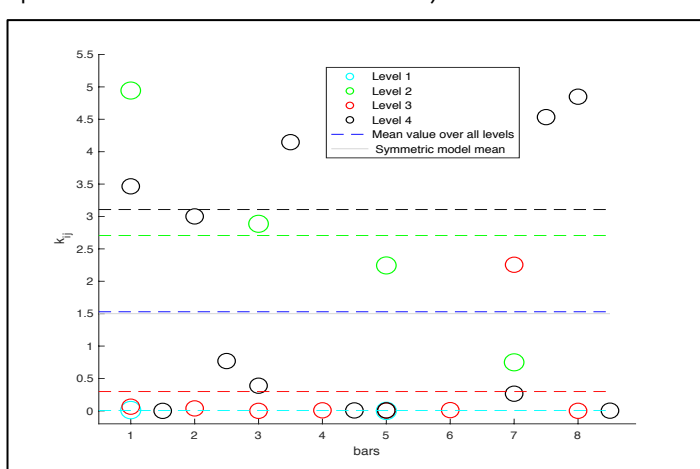
$$e_{ij} = k_{ij} * e_0 / N_i,$$

Weights coefficients k_{ij} may differ both within the specific level and over all levels.

Optimization method:

- ✓ The Nelder-Mead simplex algorithm (built-in in *Matlab*) was used to evaluate the generic model with the varying values of e_{ij} .
- ✓ Coefficients k_{ij} together with the global value e_0 were set as parameters to optimize for the `fminsearch` function so to minimize the distance between the model curve Y and the performance data.
- ✓ The values $e_0 = 0.5$ and symmetrical coefficients $k_{ij} = 1.5$ were used as initial guess for the first simplex.

Optimized coefficients k_{ij} at the levels 1–4 (colored dashed lines represent the mean values for each level):



V. Improved symmetric model

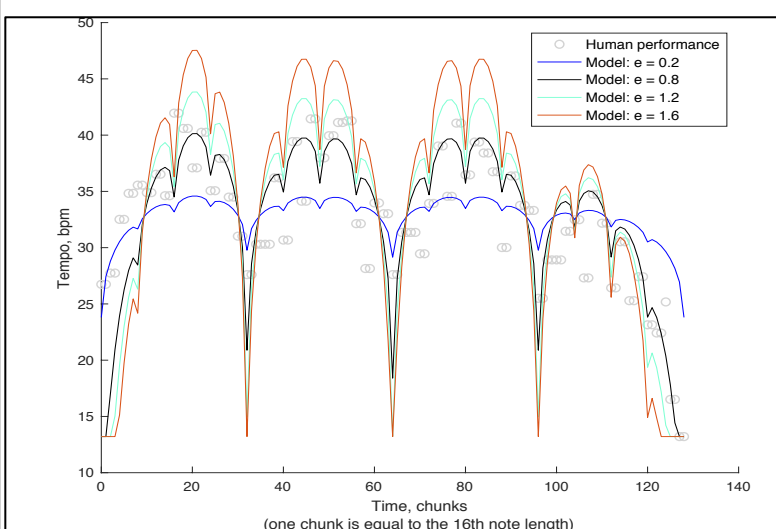
Analysis of the optimized weights distribution helps to build an improved symmetric model with the **three following assumptions**:

- Levels 2 and 4 are more elastic than the levels 1 and 3:

| Levels | k_{ij} , average value (from optimization process) | k_{ij} , improved symmetric model |
|--------|--|-------------------------------------|
| 1 | 0.006 | 0.5 |
| 2 | 2.706 | 2.5 |
| 3 | 0.298 | 0.5 |
| 4 | 3.107 | 2.5 |

- Temporal elasticity of the first ellipse at the second level (ellipse over first four bars) is boosted so to give the same elasticity as for the next four bars for emulation of performer’s expression at the beginning of the piece.
- Boundary conditions are set for the start and end tempo so to allow the more elastic phrasing.

Improved symmetric model for different values of temporal elasticity of the main arch:



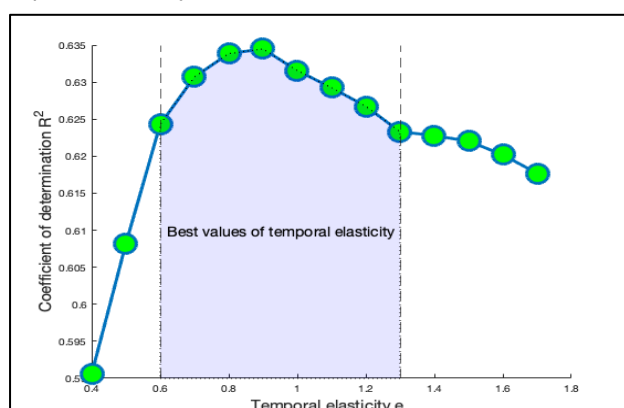
VI. Regression analysis and best values of temporal elasticity

The summary of regression analysis depending on the assumptions made:

| Number of assumptions | R^2 | Optimal value of e_0 |
|--|-------|------------------------|
| No assumptions (symmetric model) | 0.43 | 0.43 |
| One (only unequal levels) | 0.47 | 0.415 |
| Two (unequal levels and k_{21} boost) | 0.61 | 0.55 |
| Three (unequal levels, k_{21} boost and boundary conditions) | 0.63 | 0.82 |

- All coefficients R^2 are significant with $p < 0.01$.
- The improved model has a better performance, than the ‘pure’ symmetric model. Specifically, introducing the elasticity boost at the second level makes a noticeable difference. It is a meaningful finding for the performance practice illustrating how essential are the first bars of the piece (*‘well begun is half done’*).
- The values of R^2 in symmetric case can be notionally compared with the results in [5], where the highest R^2 obtained for timing from the somewhat similar symmetric model was $R^2 = 0.299$.

Coefficient of determination for improved symmetric model depending on temporal elasticity e_0 :



- The values of temporal elasticity within the interval with the highest values of R^2 are musically the most convincing expressive strategies for the performer. The less elastic phrasing might be considered as mechanical, or non-expressive, while the hyper-elastic phrasing is tasteless or grotesque.

Future work:

- Evaluating model performance through the listening tests (www.regerexperiment.com).
- Collecting more data by analyzing the model on different short late Romantic organ pieces so to approach the generalization of weighting in the improved model.

Conclusion:

- ✓ For the first time, the mathematical model of the Riemannian motivic scheme was analytically modeled and evaluated on organ music.
- ✓ The model has a two-fold application: it can be used in performance analysis, as well as in computer simulation of expressive timing.

References:

- [1] S. Popp and S. Shigihara, *Max Reger: At the Turning Point to Modernism*. Bonn: Bouvier Verlag, 1988.
- [2] J. Laukvik, “Aspekte Musikalischer Interpretation in der Spätromantik,” in *Orgelschule zur historischen Aufführungspraxis*. Stuttgart: Carus, 2006.
- [3] L. Lohmann, “Hugo Riemann and the Development of Musical Performance Practice,” in *Proc. of the Göteborg International Organ Academy*, Göteborg, 1994.
- [4] H. Riemann, *Musikalische Dynamik und Agogik, Lehrbuch der musikalischen Phrasierung auf Grund einer Revision der Lehre von der musikalischen Metrik und Rhythmik*. Hamburg, 1884.
- [5] L. Windsor and E. Clarke, “Expressive Timing and Dynamics in Real and Artificial Musical Performances: Using an Algorithm as an Analytical Tool,” *Music Perception*, vol. 15, no 2, pp. 127–152, 1997.